WEEKLY-TERM OPTIMIZATION FOR A HYDRO POWER PLANT CASCADE WITH REVERSIBLE UNITS. MATHEMATICAL MODEL

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Hydropower plants with reversible units are not only a power plant, but also a managed tool, because can take on the tasks of peak regulation, frequency modulation, phase modulation, emergency generation etc. in power grid, or inter-basin transfer of water with multiple uses. In this paper, a mathematical model for the weekly optimal scheduling of a hydropower plant cascade with reversible units is presented. The optimization problem is solved by an evolutionary method based on the genetic algorithms. The performance function take into account the price / cost of the generated consumed power, but the violation of some operational restrictions is also penalized. The operation characteristics of units in both generating and pumping modes were used.

Keywords: optimization, hydro power plant cascade, reversible units.

1. Introduction

The presence of hydropower plants that can operate both in turbinated and also in pumping regimes is very important in a power system that has several production units (gas and coal power stations, electro - nuclear power stations etc.).

Even though they cannot be compared with the role and position that pumping storage hydropower stations have within the power systems, the hydropower plant cascade with reversible units can contribute to the taking over of the power excess from the off-peak load periods and to the increase of the power generated during the peak periods.

Along with this more or less significant contribution, if the cascade system has multiple uses that involve water consumption (irrigations, water supplies for the neighbouring communities etc.), thus reducing the power capacities of the upstream inflow volumes, the respective consumptions can be provided by pumping from the water flow in which the river is discharged. Such an inter-basin

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transfer can be important if the water amounts taken for irrigations are large and the upstream inflow discharge is low.

In the present paper it is assumed that the cascade lakes have adequately large useful volumes which allow weekly operation analysis considering the differences both between the daily peak and base load level, but also between the working days and the weekend. In order to simplify, but also in order to face the lack of economic data about the non-energetic consumptions (extracted flows, selling costs and prices), the numerical applications were restricted only to the cases relevant from the energetic perspective.

The optimization model for weekly operation aims at maximizing the difference between the gains attained from the power produced in generation mode and the costs of the power consumed through pumping. The various common restrictions imposed in operation were taken into account (water levels between the allowed maximum/minimum values, the maximum level gradients accepted, the limitation of maximum turbine capacity depending on the head, allowing pumping during off-peak load hours) and the optimization model is solved through a genetic algorithm adapted to the analysed problem.

Although evolutionary algorithms (genetic algorithms, the ant colony algorithm, the particle swarm algorithm, the honey-bee mating optimization algorithm, etc.) are increasingly used in approaching water resources management problems, the authors are not aware of a bibliographic reference that is based on the above-mentioned algorithms and dedicated to this particular problem.

2. Formulation of the optimization problem

Let's assume the system of L lakes and cascade hydro power plants (HPP) as in Figure 1.

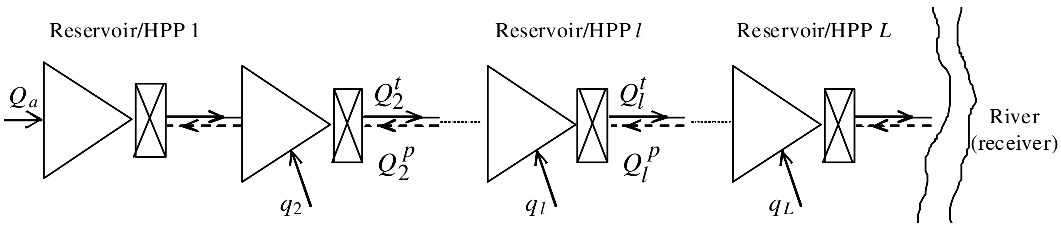

Fig. 1. Display of the cascade arrangement for the optimization model

The analysed week timeframe-horizon is divided into N time steps of Δt length, conveniently selected such as to be able to show the light/load periods and the peak periods within the K = 7 days (for example $\Delta t = 3$ hours). Let's name N_t the set of time steps in which the generation mode is admitted and N_p the set of the time steps during which the pumping is accepted.

The balance equation for a given lake l in the system at n time step, is the following:

$$V_{l,n} = V_{l,0} + \alpha\Delta t\left[\sum_{j=1}^{n} q_{l,j} + \sum_{\substack{j=1 \\ j\in N_t}}^{n}\left(Q_{l-1,j}^{t} - Q_{l,j}^{t}\right) + \sum_{\substack{j=1 \\ j\in N_p}}^{n}\left(Q_{l,j}^{p} - Q_{l-1,j}^{p}\right)\right], \qquad (1)$$

where $V_{l,0}$ is the volume in the lake at the beginning of the week; $V_{l,n}$ is the volume in the lake at the end of time step n; $q_{l,j}$ is the incoming flow in the sub-basin on step j; $Q_{l,j}^{t}$ represents the turbinated discharge by station l if $j \in N_t$; $Q_{l,j}^{p}$ represents the flow pumped by station l in its own lake if the step $j \in N_p$, and α is a size adjustment coefficient.

For the head-cascade lake instead of $q_{l,j}$ we have $Q_{a,j}$ the upstream flow, and the terms $Q_{l-1,j}^{t}$ and $Q_{l-1,j}^{p}$ disappear from relation (1).

It is admitted that on the domain between the normal retention level (NRL) and the minimum operating level (MOL), the capacity curve of each lake can be linearly approximated in the following form:

$$V_l = a_l + b_l \cdot Z_l, \qquad (2)$$

where Z_l represents the level of the free surface, and a_l and b_l are adequate coefficients.

The operating restrictions with respect to the existing volumes of the lakes are the following:

For the upstream lake:

- uniform daily emptying with ΔZ_1^* imposed amount ($\Delta V_1^* = b_1 \cdot \Delta Z_1^*$) for the first 5 working days of the week , that is:

$$V_{1,m\cdot k} = V_{1,m\cdot(k-1)} + \Delta V_1^*, \text{ for } k = 1,2,...,5, \qquad (3.1)$$

- uniform daily filling up with $\Delta Z_2^* = 5 \cdot \Delta Z_1^* / 2$ and $\Delta V_2^* = b_1 \cdot \Delta Z_2^*$ for the weekend days, that is:

$$V_{1,m\cdot k} = V_{1,m\cdot(k-1)} + \Delta V_2^* \text{, for } k = 6,7 \text{ ,} \tag{3.2}$$

where m represents the number of Δt steps in one day ($m = 24/\Delta t$).

For the other lakes in the ensemble:

- daily level variation restricted to at most ΔZ_3^* imposed value, namely:

$$\left| V_{l,m\cdot k} - V_{l,m\cdot(k-1)} \right| \le \Delta V_l \text{, for } l = 2,3,\ldots,L;\ k = 1,2,\ldots,K \text{ ,} \tag{4}$$

where $\Delta V_l = b_l \cdot \Delta Z_3^*$.

For all lakes in the system:

- positioning between the allowed minimum and maximum volumes, namely:

$$V_l^{\min} \le V_{l,n} \le V_l^{\max} \text{, for } n = 1,2,\ldots,N-1;\ l = 1,2,\ldots,L \text{,} \tag{5}$$

where $V_l^{\max}$ corresponds to the volume at NRL and $V_l^{\min}$ to the volume at MOL;

- final return to the volumes existing in the lakes at the beginning of the analysed week , namely:

$$V_{l,N} = V_{l,0} \text{, for } l = 1,2,\ldots,L \text{.} \tag{6}$$

The performance function of the system was selected as following:

$$\max\left\{ F = \sum_{l=1}^{L} \left(\sum_{\substack{n=1 \\ n \in N_t}}^{N} c_n^t \cdot E_{l,n}^t - \sum_{\substack{n=1 \\ n \in N_p}}^{N} c_n^p \cdot E_{l,n}^p \right) \right\}, \tag{7}$$

where $E_{l,n}^t$ represents the power produced during the generation mode at plant l for step $n \in N_t$, $E_{l,n}^p$ power used for pumping at plant l for step $n \in N_p$, c_n^t is the price of the power produced in generation at step n, and c_n^p is the cost of the power used for pumping at step n. Relation (7) maximizes the net income obtained from the operation of the system. If values of c_n^t and c_n^p are imposed to be equal to 1, relation (7) maximizes the net power production obtained from the system.

As far as determining the amounts of produced/consumed power is concerned, it is admitted that each station is equipped with G identical reversible units and operation characteristics are known in the form $P = f(Q, H)$ both in generation and in pumping modes, where P represents the power, Q – the flow and H the net head or the pumping head.

In generation regime, the matrix of the turbine power is specified on plausible domains for heads H and flows Q, with reasonable ΔQ and ΔH steps, and for any set of Q and H values, power P is computed by bi-dimensional linear interpolation. In addition, some regression equations for the technical minimum flow, $Q_{th}^{\min}(H)$, and for the maximum flow, $Q^{\max}(H)$ are defined versus head H, and these limits are taken into account for power determinations.

Head losses in the plant Δh_c for *flow Q*, are estimated if the values Δh_{inst} are known for the installed flow of the unit Q_{inst}, by the relation:

$$\Delta h_c(Q) = \Delta h_{inst}(Q_{inst}) \cdot \left(\frac{Q}{Q_{inst}} \right)^2, \tag{8}$$

and the downstream head reduction is obtained from hydraulic calculations as function of the overall turbinated flow, Q^t, in the form $\Delta h_{av} = f_1(Q^t)$. In this way the net head of plant l is obtained with the relation:

$$H_l^t = Z_l - Z_{l+1} - \Delta h_c(Q) - \Delta h_{av}(Q_l^t), \tag{9}$$

where $Q = Q_l^t / n_l^t$ is the used flow for each of the $n_l^t \le G$ equally loaded units and Z_l and Z_{l+1} are the free surfaces levels of the associated lake and the downstream lake, respectively, in static conditions.

In pumping mode, since the domain guaranteed in operation is relatively narrow, a linear variation versus the head H, both for the pumped flow and also for the absorbed power can be admitted. The head loss in the station was also estimated by using relation (8), and the downstream head increase versus the overall pumped flow, Q^p, is known from hydraulic calculation in the form: $\Delta h_s = f_2(Q^p)$.

The overall pumping head at plant l will then be:

$$H_l^p = Z_l - Z_{l+1} + \Delta h_c(Q) + \Delta h_s\left(Q_l^p\right), \tag{10}$$

where $Q = Q_l^p / n_l^p$ is the flow pumped on each of the $n_l^p \le G$ pumping units.

At computation on successive time steps, Z_l levels will be considered as average values on each Δt time step. In addition, as the minimum technical flow, the maximum turbinated flow and the pumped flow depend on the head, they will be initially evaluated for the conditions existing at the beginning of the first time step and then they will be adjusted through a number of iterations on the respective step.

It is obvious that the power obtained in generation and for pumping mode will be next adjusted with the generator efficiency and the motor efficiency, respectively, which were here assumed as constant values for simplification reasons.

As a conclusion, if the sets of turbined flows $Q_{l,j}^t$, for $l = 1,2,...,L$, $j \in N_t$ and the pumped flows, respectively, $Q_{l,j}^p$, for $l = 1,2,...,L$, $j \in N_p$, are known, the evolution of the system during the considered time horizon can be determined, the produced/consumed power can be computed and the performance function can be assessed for the mentioned set of values. The problem yet to solve is the finding of the set of values $Q_{l,j}^t$ and $Q_{l,j}^p$ which can observe the imposed restrictions for the volumes (water levels), can be admissible as equipment capabilities and can lead to the maximization of the performance function.

3. Operation optimization by a genetic algorithm model (AG)

An optimization problem such as the one briefly summarized above belongs to the class of the *large size, non linear and hard problems*. They cannot be efficiently approached, or they cannot be approached at all, via classical mathematical programming methods.

However, the probabilistic searching algorithms that have recently been developed allow satisfying suboptimal solutions without major computation difficulties for such problems. We can name here the simulated annealing algorithm but also the evolutionary algorithms, as the ant colony algorithm and the particle swarm algorithm. A sufficiently detailed description of what genetic algorithms are and how they operate can be found for example in [1], [2], therefore only adjustment particularities to the considered problem will be presented here.

It is known that AG operates with populations of solutions. In this paper a solution is formed by $L \times N$ numerical values. For each lake l, $l = 1,2,...,L$, N such values correspond, and they are represented by $Q_{l,j}^{t}$ the turbinated flows for $j \in N_t$ and by $n_{l,j}^{p}$ the number of pumping units, for $j \in N_p$, respectively.

For each solution of the initial population, values in the domain $[0; G \cdot Q_{inst}]$ are randomly generated for $Q_{l,j}^{t}$ turbinated flows, and integer numbers in the domain $[0; G]$ are generated for $n_{l,j}^{p}$. These values are fed to a simulation model for the weekly operation of the system. The model is iteratively runned in order to correct the possible values of the decision variables that lead to breaking certain restrictions (overtaking the allowed maximum or minimum volumes, overtaking the imposed daily level gradients, overtaking the turbinated capacity at the resulting head etc.) and in order to assess the elements necessary in power calculations.

In the end the value of the performance function is computed for each solution with a relation as in (7), but two penalty factors are subtracted, namely:

$$p_1 \sum_{k=1}^{K} \left(V_{1,m \cdot k} - V_{1,m \cdot k}^{*} \right) \text{ and } p_2 \sum_{l=1}^{L} \left(V_{l,N} - V_{l,0} \right). \tag{11}$$

The first term refers to observing the daily emptying/filling up programme for the upstream lake and the second term refers to the lake volumes returning to their initial values. Very high arbitrary values are adopted for p_1 and p_2 and $V_{1,m \cdot k}^{*}$ represents the volumes computed by relations (3.1) and (3.2).

Further, the manner for acting upon the possible corrections of the decision variables is exemplified with reference to the computation algorithm implemented in the operation simulation model. If in any time step of the generation period, the value of the turbinated flow at the plant associated to a lake goes beyond the maximum allowed volume, the algorithm increases that flow but without exceeding the turbine capacity for the existing head. When this increase is not enough to observe the volume restriction, the algorithm lowers the turbinated flow on the corresponding time step in the upstream plant. If the time step corresponds to a pumping period, the number of pumping units at that plant is reduced successively, and if even after stopping all units the maximum allowed volume is still exceeded, the algorithm increases successively the number of units that pump in the upstream plant until the volume restriction is observed.

Naturally, at lowering below the minimum allowed volume, the algorithm operates inversely, both in generation and in pumping modes. If, when lowering the turbinated flow this ends up being lower than the minimum technical flow on the unit, the turbinated flow will be imposed zero at the plant associated to the considered lake, and the correction for observing the volume restriction is made on account of the upstream plant. The corrections that lead to observing restrictions (4) are performed in a similar manner.

Each solution of the initial population is treated as above (and it is adjusted such as to observe some of the important restrictions of the problem) and one value of the performance function is obtained for each solution.

AG creates a new generation of solutions starting from the initial population.

Two solutions from the previous population are selected by a random selection process – but which gives higher chances to more performing solutions (here the selection by normalized geometric ranking was used). These two parent solutions are combined by the arithmetic crossover operator (applied with 0.75 probability) and are altered through a non – uniform mutation operator (applied with 0.02 probability) – in order to produce two new solutions (children solutions) which are expected to have better values of the performances functions. The selection, crossover and mutation operations are repeated until completing the number of solutions that are necessary in the new generation. In evaluating each new solution, the simulation model algorithm makes the decision variables correction, if necessary, for observing the above- mentioned restrictions.

After creating and evaluating the solutions of the current generation, the described process is repeated in order to produce a new generation, and so on.

Newer and newer generations are thus created, and the value of the performance function of the best solution is gradually ameliorated. Moreover, the generation average of the performance function is improved, the solutions of a given generation becoming increasingly closer and better.

AG can stop either after going through a pre-established number of generations or in accordance with a different conveniently selected criterion.

R E F E R E N C E S

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