

THE INFLUENCE OF VELOCITY VARIABILITY ON POLLUTANT DISPERSION IN GROUNDWATER

Anca Marina MARINOV¹, Mihaela Amalia DIMINESCU²

We will consider the dispersion of a conservative pollutant into an unconfined aquifer. The contaminant source is a polluted lake which is a boundary for the groundwater.

Considering the continuity equation and the Darcy's law for an unconfined aquifer, the velocity and the water table level will be computed, related with the boundary conditions. The water table level $h(x)$ is obtained from the steady-state equation of one-dimensional flow through a saturated, homogenous, isotropic aquifer. The unconfined aquifer is limited by two lakes whose levels and qualities determine the flow in the aquifer and the boundary conditions for the pollutant dispersion. The phreatic velocity is variable in x -direction. The variable aquifer's velocity will determine a variable dispersion coefficient along the aquifer.

We propose a numerical solution of the one-dimensional dispersion equation, with variable coefficients and we analyze the difference between that one and the solution obtained for the dispersion equation with constant coefficients.

Keywords: groundwater, pollution, advection, dispersion, unconfined aquifer.

1. Introduction

We analyze the influence of a polluted lake or stream on the water quality of an unconfined neighboring aquifer. Due to the interconnections between streams and aquifers, stream pollution may influence aquifer pollution and conversely. A stream is a boundary condition in piezometric head for hydraulic model and in pollution concentration and flux for the groundwater pollution model. These concentration conditions are the outputs of a stream-pollution model. We try to obtain a simple, robust model to predict the pollutant concentration in time and space. The groundwater variable velocity is used in the dispersion-advection equation of the pollutant.

2. Hydraulic model of steady groundwater flow in unconfined aquifer

Let consider the groundwater flow in x -direction of an unconfined aquifer.

¹ Prof., Hydraulics & Hydraulic Machinery Dept, University "Politehnica" of Bucharest, Romania

² Lecturer, Hydraulics & Hydraulic Machinery Dept, University "Politehnica" of Bucharest

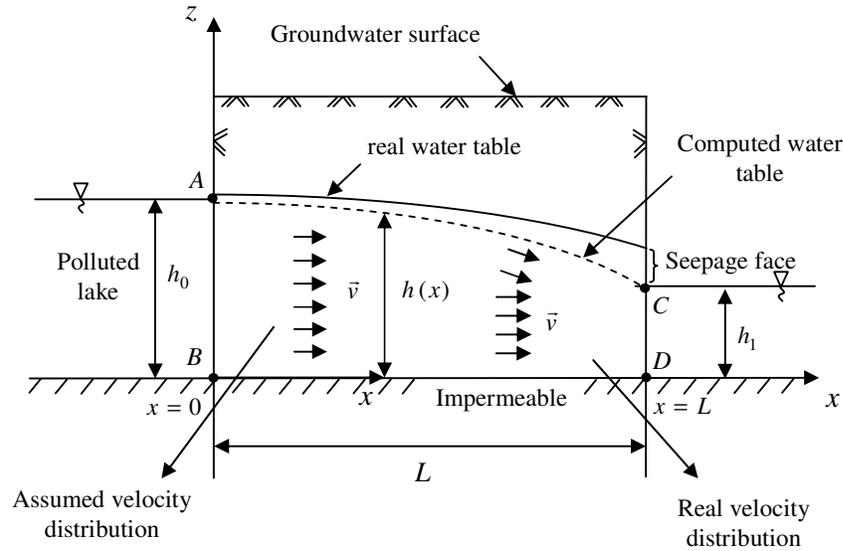


Fig. 1. Steady flow in an unconfined aquifer, between two water bodies with vertical boundaries.

For a two dimensional homogenous, isotropic medium ($K_x = K_y = K$), Darcy's law assumes the velocity in the aquifer has the components:

$$U_x = -K \frac{\partial h}{\partial x}, \quad (1)$$

$$U_z = -K \frac{\partial h}{\partial z}, \quad (2)$$

where κ [LT^{-1}] is the hydraulic conductivity, h [L] is the height of the water table above an impervious base, x [L] is the direction of flow, and z [L] is the vertical direction.

$$\vec{U} = U_x \vec{i} + U_z \vec{k} \quad (3)$$

is called Darcy velocity. The real interstitial water velocity in the groundwater is:

$$\vec{v} = \frac{\vec{U}}{n_e} \quad (4)$$

where n_e is the effective porosity of the aquifer.

In our model we use Dupuit's hypothesis: (i1) the velocity of the flow is proportional to the tangent of the hydraulic gradient instead of the sine, as defined Darcy's law; (i2) the flow is horizontal and uniform everywhere in a vertical section (no flow in z direction, $\frac{\partial h}{\partial z} = 0$, $U_z = 0$).

The Darcy's velocity in the aquifer is

$$U_x = -K \frac{dh}{dx} \quad (5)$$

and the discharge per unit width, at any vertical section is

$$q = -h \cdot K \frac{dh}{dx}. \quad (6)$$

Considering the aquifer from Fig. 1, the integral

$$\int_0^L q \, dx = \int_{h_0}^{h_1} -K h \, dh \quad (7)$$

gives the value of the discharge in the aquifer:

$$q = \frac{K}{2L} (h_0^2 - h_1^2), \quad (8)$$

h_0 and h_1 are the water levels in the two water bodies neighboring the aquifer.

The equation

$$\int_0^x q \, dx = \int_{h_0}^h -K h \, dh, \quad (9)$$

gives the thickness of the aquifer

$$h(x) = \sqrt{h_0^2 - \frac{2q}{K} x}. \quad (10)$$

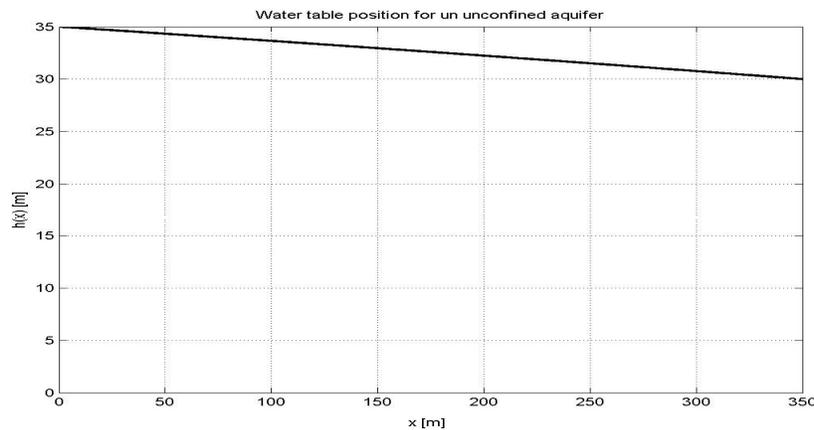


Fig. 2. Water table position for an unconfined aquifer.

The real interstitial velocity is:

$$v_x(x) = \frac{U_x(x)}{n_e} = \frac{q}{n_e \sqrt{h_0^2 - \frac{2q}{K} x}}. \quad (11)$$

The real water table does not follow the parabolic form never the less for flat slopes, where the sine and tangent are nearly equal. The equation (8) accurately determines the discharges. The water table level $h(x)$ is sufficiently correct, except near the outflow. The calculated velocity is constant for each vertical section, normal to flow, but change its value for different x . The real velocity vector varies in a vertical section, in the water table proximity and near the outflow (seepage face). Excepting the boundaries AB and DC the results obtained with Dupuit's hypothesis are general accepted.

For a particular aquifer, with $h_0 = 35$ m, $h_1 = 30$ m, $K = 30$ m/day, $n_e = 0.2$, $L = 350$ m, the water table position (10) and the velocity (11) are plotted (Figs 2 and 3).

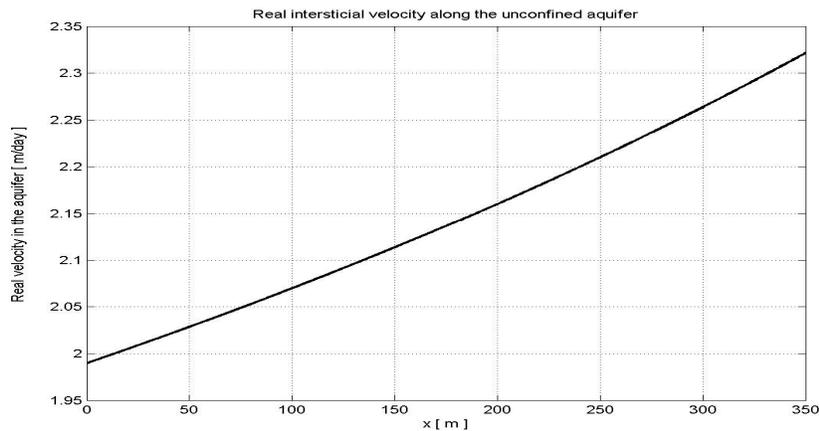


Fig. 3. Real velocity along the unconfined aquifer.

3. Mass transport of conservative pollutants in an unconfined aquifer

The dispersion is a non steady irreversible mixing process spreading the tracer within the surrounding flow system. Dispersion is essentially a microscopic phenomenon caused by a combination of molecular diffusion and hydrodynamic mixing occurring with laminar flow through porous media.

The mathematical description of dispersion is based on statistical concepts [1], on theoretical studies [3] or on experimental studies.

The law of mass conversion for solute transport in saturated media considers the flux of solute into and out of a fixed elemental volume.

Conservative solutes are non-reactive with the soil and with the groundwater and do not undergo biological or radioactive decay. For a non-

reactive dissolved substance the flux into the element and the flux out of the element are equal to the net rate of change of mass of solute within the element.

The one-dimensional, advection-dispersion equation is:

$$\frac{\partial(n_e C)}{\partial t} = \underbrace{\frac{\partial}{\partial x} \left(n_e D_x \frac{\partial C}{\partial x} \right)}_{\text{dispersion}} - \underbrace{\frac{\partial}{\partial x} (n_e v_x C)}_{\text{advection}}. \quad (12)$$

Advection is the transport of solute by the following groundwater. Hydrodynamic dispersion results from mechanical mixing (dispersivity) and molecular diffusion. $C(x, t)$ [ML^{-3}] is the pollutant concentration, $v_x(x)$ [LT^{-1}] is the average interstitial groundwater velocity, n_e is the effective porosity (dimensionless), $D_x(x)$ [$\text{L}^2 \text{T}^{-1}$] is the dispersion coefficient:

$$D_x(x) = \alpha_x v_x(x) + D, \quad (13)$$

α_x [L] is the dynamic dispersivity (a characteristic property of the porous medium), D [$\text{L}^2 \text{T}^{-1}$] is the molecular diffusion (effective diffusion coefficient). Usually $D \ll \alpha_x v_x$ (mechanical dispersion), and

$$\alpha_x = 0,1L, [3], \quad (14)$$

where L is the length of the flowpath.

Our problem is to solve the advection-dispersion equation (12) for the unconfined aquifer from (Fig. 1), considering the variable velocity given by (11), and a mechanical dispersion in the porous medium $D_x(x) = \alpha_x v_x(x)$.

For boundaries conditions we consider the polluted lake (AB) having a constant C_0 concentration, and at the outflow lake (CD) an unknown concentration. The initial values of concentration in the aquifer are known.

4. A numerical solution of the advection-dispersion equation with variable coefficients

We have obtained a numerical solution of the advection-dispersion equation (12) with initial conditions and boundaries conditions described above.

For the approximation of the one dimensional dispersion equation with variable coefficients (12) we use an implicit numerical scheme. The implicit approximations are unconditionally stable and convergent [1].

$C(x, t) = C(i, j)$ is the pollutant concentration, in aquifer, at a distance x from the lake, after t hours from the beginning of contamination ($t_0 = 0$).

In the approximation of $\partial C / \partial t$ we consider known the concentration values $C(x, t_1)$ and we are looking for $C(x, t_2)$, with $t_2 > t_1$:

$$\frac{\partial C(x,t)}{\partial t} = \frac{C(x,t_2) - C(x,t_1)}{t_2 - t_1}. \quad (15)$$

An implicit approximation implies to compute the right terms of the equation (12) at the t_2 moment.

$$\frac{C(x,t_2) - C(x,t_1)}{t_2 - t_1} = \frac{\partial}{\partial x} \left[D_x(x) \frac{\partial C(x,t_2)}{\partial x} \right] - \frac{\partial}{\partial x} [v_x(x) \cdot C(x,t_2)] \quad (16)$$

All $C(x,t_2)$ are unknown. For a constant spatial $dx = x(i+1) - x(i)$, ($i = 1 : N - 1$), and a temporal $dt = t(j+1) - t(j)$, ($j = 1 : M - 1$), the approximated fully implicit form of the equation (12) will be, for $i = 2 : N - 1$ and $j = 1 : M - 1$,
 $C(i-1, j+1) \cdot a(i, i-1) + C(i, j+1) \cdot a(i, i) + C(i-1, j+1) \cdot a(i, i+1) = l(i, j+1)$ (17)
 where

$$a(i, i-1) = \frac{-D_x(i+1) + D_x(i-1)}{4 dx^2} + \frac{D_x(i)}{dx^2} + \frac{v_x(i)}{2 dx} \quad (18)$$

$$a(i, i) = -\frac{2 D_x(i)}{dx^2} - \frac{v_x(i+1) - v_x(i-1)}{2 dx} - \frac{1}{dt} \quad (19)$$

$$a(i, i+1) = \frac{D_x(i+1) - D_x(i-1)}{4 dx^2} + \frac{D_x(i)}{dx^2} - \frac{v_x(i)}{2 dx} \quad (20)$$

$$l(i, j+1) = C(i, j) \left(-\frac{1}{dt} \right) \quad (21)$$

The boundary conditions give two equations for $i = 1$, ($x = 0$) and $i = N$, ($x = L$). A constant concentration in the polluted lake $C(x = 0, t) = C_0$ implies

$$C(1, j+1) = C_0 \text{ for } j = 1 : M - 1. \quad (22)$$

For $x = L$ we impose the condition

$$C(N-1, j+1) = C(N, j+1) \text{ for } j = 1 : M - 1 \quad (23)$$

The initial conditions are:

$$C(i, 1) = 0 \text{ for } i = 2 : N \quad (24)$$

$$C(1, 1) = C_0 \quad (25)$$

For each moment ($j+1$), ($j = 1 : M - 1$), a linear system of equation:

$$[a]\{C\} = \{l\} \quad (26)$$

is obtained, and a procedure has to be chosen to solve it.

5. Results and conclusions

We compare the results for variable velocity in the aquifer (Fig.4, Fig.5) with the solution obtained for a constant velocity [2]. A relative, procentual error is calculated (Table 1):

$$\varepsilon(\%) = (abs(C_{v\text{ var}} - C_{v\text{ const}}) / C_0) \cdot 100, \quad (27)$$

$C_{v\text{ var}}$ is the matrix of $C(i, j)$, ($i = 1 : N, j = 1 : M$) obtained for a variable velocity in the aquifer, $C_{v\text{ const}}$ is the matrix of $C(i, j)$ for a constant velocity.

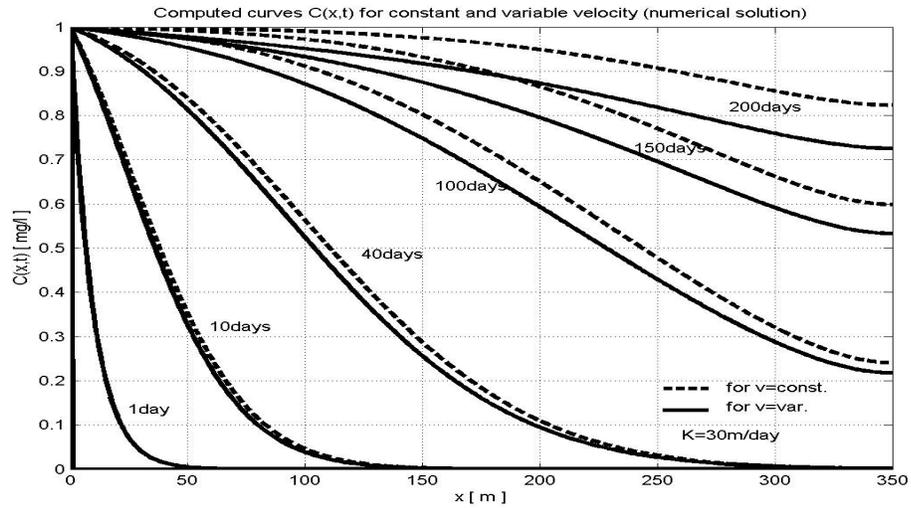


Fig. 4. Concentration $C(x,t)$, along the aquifer, at different moments, t .

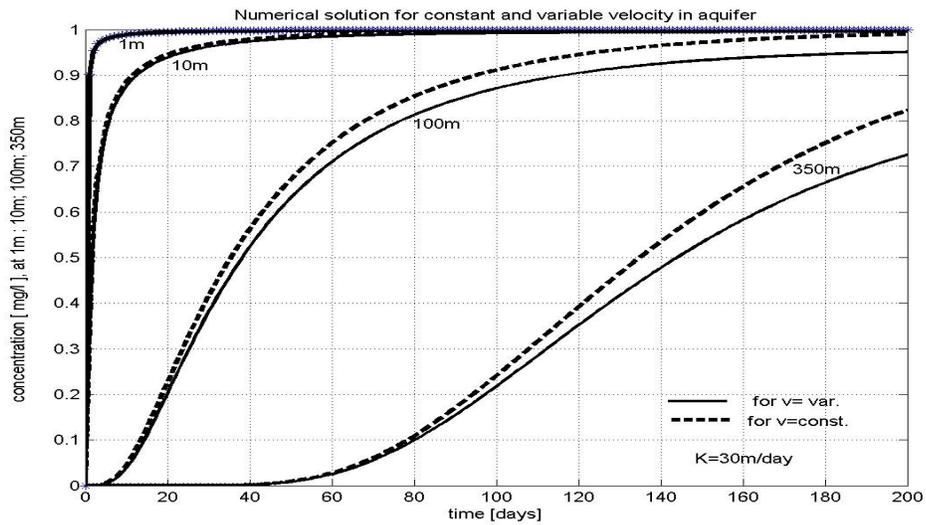


Fig. 5. Time variation of concentration $C(x,t)$, at different distances along the aquifer.

Table 1

 \mathcal{E} (%) error values for different hydraulic conductivity in the aquifer

	$t(\text{days})$ $x(\text{m})$	1	10	20	40	100	150	200
$K=30\text{m/day}$	1	0.3989	0.0894	0.0649	0.0498	0.0406	0.0393	0.0390
	10	1.5266	0.9133	0.6678	0.5078	0.4077	0.3942	0.3909
	100	0.0010	0.7617	2.4931	3.9051	4.0320	3.9604	3.9503
	150	0.0000	0.0689	0.8014	3.1094	5.4131	5.7339	5.8694
	200	0.0000	0.0032	0.1284	1.4974	5.7158	7.0040	7.5774
	350	0.0000	0.0000	0.0000	0.0147	2.3090	6.5458	9.7852
$K=10\text{m/day}$	1	0.6300	0.1500	0.1100	0.0800	0.0500	0.0500	0.0400
	10	1.1100	1.4300	1.0700	0.7900	0.5400	0.4700	0.4400
	100	0.0000	0.0100	0.2000	1.4000	3.7100	4.0800	4.1200
	150	0.0000	0.0000	0.0000	0.1700	2.4400	3.9500	4.7600
	200	0.0000	0.0000	0.0000	0.0100	0.8700	2.4600	3.9000
	350	0.0000	0.0000	0.0000	0.0000	0.0600	0.0600	0.3700
$K=1\text{m/day}$	1	1.2200	0.4500	0.3200	0.2200	0.1400	0.1200	0.1000
	10	0.0500	1.5900	2.0000	1.8700	1.4000	1.1900	1.0600
	100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0400	0.1700
	150	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	200	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	350	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

The difference between the computed curves $C(x,t)$ for constant velocity and variable velocity is significant, after 40 days, for a distance $x \geq 100\text{m}$.

The concentration values computed considering a variable velocity in the aquifer are generally less than those calculated with a constant velocity. The maxim difference is obtained for a $x = 350\text{m}$ distance, after 40 days.

Our conclusion is the more accurate results are obtained considering a variable velocity in the unconfined aquifer. The calculated error \mathcal{E} decreases with K value. For a hydraulic conductivity $K = 1\text{ m/day}$ the differences \mathcal{E} are insignificant so for this case a constant velocity value can be used to compute the concentrations.

REFERENCES

- [1]. *J. J. Freid*, Groundwater Pollution, Elsevier, 1975.
 [2]. *Anca Marina Marinov*, "Modelarea matematică a poluării apelor subterane în zona Curtișoara-Slatina", in *Lucr. celei de-a IV-a Conferințe a Hidroenergeticienilor din România "Dorin Pavel"*, București, mai 2006.
 [3]. *D. K. Todd and L. W. Mays*, Groundwater Hydrology, John, Wiley and Sons, Inc., Third Edition, 2005.