SURFACE AERATOR FOR WASTE WATER TREATMENT

Mircea Dimitrie CAZACU[1*], Iuliana Monica BOIANGIU[2]

One presents the theoretical research of a patented biphasic contactor, that realize a very efficient contact between a liquid and a gas, controlling the liquid bubble dimension by the rotations number of an immersed cone and the equilibrium between the inertia and molecular attraction forces.

For the numerical solving of the viscous and heavy liquid film flow, under the action of the centrifugal and molecular attraction forces, exerted by the immersed turning cone, we realized the equations transformation from the co-ordinates system attached to the peak of the submerged cone, to this placed on the cone generatrix.

Keywords: surface aerator, biphasic contactor, controlled dimension bubbles, waste water treatment, molecular attraction forces.

1. Introduction

Our theoretical research concerning the patented biphasic rotational contactor [1] began in 1996 year [2][3][4][5] and consisted in the transformation of the viscous liquid partial differential equations from the cylindrical coordinates 0,R,θ,Z in the Cartesian coordinates 0',X,Y, ζ of the layer flow adjacent to the cone generatrix (figure 1), while the experimental research occupied with the determination of its important performances.

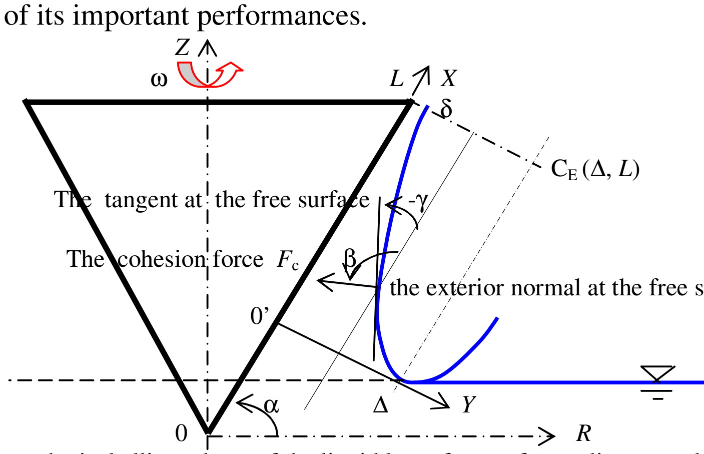

Fig.1. The hypothetical ellipse shape of the liquid layer free surface, adjacent to the cone

[1] Prof., Power Engineering Faculty, Hydraulics and Hydraulic Machinery Department, University "Politehnica" of Bucharest, Romania
[2] PhD Student, Engineer, Trading Company CONFORT Ltd., Bucharest, Romania

In this paper we shall present the finale theoretical part of the viscous liquid flow equation transformation for the fluid motion in this layer, generated by the turning round of the cone immersed in the liquid and working on the basis of molecular attraction (of adhesion and cohesion) and inertia forces, having the possibility to control the size of the liquid bubbles by change of its rotation number.

2. The system of the equations with partial differentials

The obtained equation system is composed of the three motion equations:

$$\begin{aligned}&\left(U'_{X}\cos\alpha+V'_{X}\sin\alpha\right)U+\left(U'_{Y}\cos\alpha+V'_{Y}\sin\alpha\right)V-\frac{W^2}{R_0+X\cos\alpha+Y\sin\alpha}+\\&+\frac{P'_{X}\cos\alpha+P'_{Y}\sin\alpha}{\rho}=\upsilon\left[\begin{array}{l}\left(U''_{X^2}+U''_{Y^2}\right)\cos\alpha+\left(V''_{X^2}+V''_{Y^2}\right)\sin\alpha-\\-\dfrac{U\cos\alpha+V\sin\alpha}{\left(R_0+X\cos\alpha+Y\sin\alpha\right)^2}+\\+\dfrac{U'_{X}\cos^2\alpha+\left(V'_{X}+U'_{Y}\right)\sin\alpha\cos\alpha+V'_{Y}\sin^2\alpha}{R_0+X\cos\alpha+Y\sin\alpha}\end{array}\right]-\\&-\left\{F_{a}\left(1-\frac{Y}{\Delta}\right)\sin\alpha\right\}_{D_2}+\left\{F_{c}\left(1-\frac{Y}{\delta}\right)\sin\alpha\right\}_{D_3}+\left\{F_{c}\left(1-\frac{d}{\delta}\right)\cos\left[\alpha+\beta\left(X_{E},Y_{E}\right)\right]\right\}_{D_4},\end{aligned}\tag{1}$$

$$\begin{aligned}&W'_{X}U+W'_{Y}V+\frac{\left(U\cos\alpha+V\sin\alpha\right)W}{R_0+X\cos\alpha+Y\sin\alpha}=\\&=\upsilon\left[W''_{X^2}+W''_{Y^2}+\frac{W'_{X}\cos\alpha+W'_{Y}\sin\alpha}{R_0+X\cos\alpha+Y\sin\alpha}-\frac{W}{\left(R_0+X\cos\alpha+Y\sin\alpha\right)^2}\right],\end{aligned}\tag{2}$$

$$\begin{aligned}&\left(U'_{X}\sin\alpha-V'_{X}\cos\alpha\right)U+\left(U'_{Y}\sin\alpha-V'_{Y}\cos\alpha\right)V+\frac{P'_{X}\sin\alpha-P'_{Y}\cos\alpha}{\rho}+g=\\&=\upsilon\left[\begin{array}{l}\left(U''_{X^2}+U''_{Y^2}\right)\sin\alpha-\left(V''_{X^2}+V''_{Y^2}\right)\cos\alpha+\\+\dfrac{U'_{Y}\sin^2\alpha+\left(U'_{X}-V'_{Y}\right)\sin\alpha\cos\alpha-V'_{X}\cos^2\alpha}{R_0+X\cos\alpha+Y\sin\alpha}\end{array}\right]+\\&+\left\{F_{a}\left(1-\frac{Y}{\Delta}\right)\cos\alpha\right\}_{D_2}-\left\{F_{c}\left(1-\frac{Y}{\delta}\right)\cos\alpha\right\}_{D_3}+\left\{F_{c}\left(1-\frac{d}{\delta}\right)\sin\left[\alpha+\beta\left(X_{E},Y_{E}\right)\right]\right\}_{D_4},\end{aligned}\tag{3}$$

and the mass conservation equation

$$U'_{X}+V'_{Y}+\frac{U\cos\alpha+V\sin\alpha}{R_o+X\cos\alpha+Y\sin\alpha}=0.\tag{4}$$

Having in view the combined mode in which appear the pressure partial differentials in the two motion equations (1) and (3), uncomfortable to eliminate the continuous, uniform and bounded pressure function in virtue of Schwarz's commutativity relation $P''_{XY} = P''_{YX}$ of the 2nd order mixed derivative, we shall perform the following preliminary transformation. By multiplying with cos α the equation (1) and with sin α the equation (3), we shall obtain by their addition

$$U'_X U + U'_Y V - \frac{W^2 \cos\alpha}{R_0 + X\cos\alpha + Y\sin\alpha} + \frac{1}{\rho} P'_X + g\sin\alpha =$$
$$= \upsilon\left[U''_{X^2} + U''_{Y^2} - \frac{\cos\alpha\left(U\cos\alpha + V\sin\alpha\right)}{\left(R_0 + X\cos\alpha + Y\sin\alpha\right)^2} + \frac{U'_X\cos\alpha + U'_Y\sin\alpha}{R_0 + X\cos\alpha + Y\sin\alpha}\right] + \tag{5}$$
$$+\left\{F_c\left(1 - \frac{\sqrt{\left(X - X_E\right)^2 + \left(Y - Y_E\right)^2}}{\delta}\right)\cos\left[\beta\left(X_E, Y_E\right)\right]\right\}_{D_4} ,$$

and multiplying with sin α the equation (1) and with cos α the equation (3), we shall obtain by their subtraction

$$V'_X U + V'_Y V - \frac{W^2 \sin\alpha}{R_0 + X\cos\alpha + Y\sin\alpha} + \frac{1}{\rho} P'_Y - g\cos\alpha =$$
$$= \upsilon\left[V''_{X^2} + V''_{Y^2} - \frac{\sin\alpha\left(U\cos\alpha + V\sin\alpha\right)}{\left(R_0 + X\cos\alpha + Y\sin\alpha\right)^2} + \frac{V'_X\cos\alpha + V'_Y\sin\alpha}{R_0 + X\cos\alpha + Y\sin\alpha}\right] - \tag{6}$$
$$-\left\{F_a\left(1 - \frac{Y}{\Delta}\right)\right\}_{D_2} + \left\{F_c\left(1 - \frac{Y}{\delta}\right)\right\}_{D_3} - \left\{F_c\left(1 - \frac{\sqrt{\left(X - X_E\right)^2 + \left(Y - Y_E\right)^2}}{\delta}\right)\sin\left[\beta\left(X_E, Y_E\right)\right]\right\}_{D_4} .$$

obtaining by pressure function elimination, the equation

$$U\left(U''_{XY} - V''_{X^2}\right) + \left(U'_Y - V'_X\right)\left(U'_X + V'_Y\right) + V\left(U''_{Y^2} - V''_{XY}\right) + 2W\frac{W'_X\sin\alpha - W'_Y\cos\alpha}{R_0 + X\cos\alpha + Y\sin\alpha} =$$
$$= \upsilon\left(U'''_{X^2Y} + U'''_{Y^3} - V'''_{X^3} - V'''_{XY^2} + \frac{V'_X - U'_Y}{R^2} + \frac{U''_{XY}\cos\alpha + U''_{Y^2}\sin\alpha - V''_{X^2}\cos\alpha - V''_{XY}\sin\alpha}{R_0 + X\cos\alpha + Y\sin\alpha}\right) -$$
$$-F_c\left\{\begin{aligned}&\left(1 - \frac{d}{\delta}\right)\beta'_Y\sin\beta\left(X_E, Y_E\right) + \frac{Y - Y_E}{\delta d}\cos\beta\left(X_E, Y_E\right) + \\ &+\left(1 - \frac{d}{\delta}\right)\beta'_X\cos\beta\left(X_E, Y_E\right) - \frac{X - X_E}{\delta d}\sin\beta\left(X_E, Y_E\right)\end{aligned}\right\}_{D_4} , \tag{7}$$

3. The approximation of the liquid free surface curve with an ellipse

In this case, from the ellipse equation with the center in the point $C_E\,(L,\Delta)$

$$\left(X_E - L\right)^2 / L^2 + \left(Y_E - \Delta\right)^2 / \left(\Delta - \delta\right)^2 = 1\,, \tag{8}$$

we deduced from the 2[nd] order equation

$$Y_E^2 - 2\Delta Y_E + \Delta^2 + \left(\Delta - \delta\right)^2 \left(\frac{X_E^2}{L^2} - 2\frac{X_E}{L} \right) = F\left(X_E, Y_E\right) = 0\,, \tag{8'}$$

taking into consideration that, although the both solutions are real the quantity under the radical being always positive $\sqrt{2X_E/L - X_E^2/L^2}$ for $0 \le X_E/L \le 1$, only the solution with negative mark is convenient to the problem, la abscisa $X = L$, unde pelicula de lichid părăseşte muchia periferică a conului.

Calculating the partial differentials:

$$F'_{X_E} = 2\left(X_E - L\right)\left(\Delta - \delta\right)^2 / L^2 \text{ and } F'_{Y_E} = 2\left(Y_E - \Delta\right), \tag{9}$$

of the function $F(X_E, Y_E)$ that defines the free surface ellipse, the tangent to the ellipse may be written in function of its angle γ or fo normale angle β

$$\frac{\mathrm{d}Y_E}{\mathrm{d}X_E} = \frac{F'_{X_E}}{F'_{Y_E}} = \frac{X_E - L}{Y_E - \Delta}\left(\frac{\Delta - \delta}{L}\right)^2 = \mathrm{tg}\,\gamma = \mathrm{tg}\left(\beta - \frac{\pi}{2}\right) = \frac{-1}{\mathrm{tg}\,\beta}\,, \tag{10}$$

from where we deduce the expressions:

$$\mathrm{tg}\,\beta = -\frac{\left(Y_E - \Delta\right)}{\left(X_E - L\right)}\frac{L^2}{\left(\Delta - \delta\right)^2} \quad \rightarrow \quad \beta\left(X_E, Y_E\right) = \mathrm{arc\,tg}\left[-\frac{\left(Y_E - \Delta\right)L^2}{\left(X_E - L\right)\left(\Delta - \delta\right)^2} \right], \tag{11}$$

as well as the one's of the argument partial differentials:

$$\beta'_{X_E} = \frac{\left(\Delta - \delta\right)^2 \left(Y_E - \Delta\right) L^2}{\left(X_E - L\right)^2 \left(\Delta - \delta\right)^4 + \left(Y_E - \Delta\right)^2 L^4}\,, \quad \beta'_{Y_E} = \frac{-L^2\left(X_E - L\right)\left(\Delta - \delta\right)^2}{\left(X_E - L\right)^2 \left(\Delta - \delta\right)^4 + \left(Y_E - \Delta\right)^2 L^4}\,.$$

With these expressions the equation (7) becomes finally:

$$U\left(U''_{XY} - V''_{X^2}\right) + \left(U'_Y - V'_X\right)\left(U'_X + V'_Y\right) + V\left(U''_{Y^2} - V''_{XY}\right) + 2W\frac{W'_X \sin\alpha - W'_Y \cos\alpha}{R_0 + X\cos\alpha + Y\sin\alpha} =$$

$$= \upsilon\left(U'''_{X^2Y} + U'''_{Y^3} - V'''_{X^3} - V'''_{XY^2} + \frac{V'_X - U'_Y}{R^2} + \frac{U''_{XY}\cos\alpha + U''_{Y^2}\sin\alpha - V''_{X^2}\cos\alpha - V''_{XY}\sin\alpha}{R_0 + X\cos\alpha + Y\sin\alpha} \right) -$$

$$-F_c \left\{ \begin{aligned} &\left(1 - \frac{d}{\delta}\right)\left(\frac{-L^2\left(X_E - L\right)\left(\Delta - \delta\right)^2}{\left(X_E - L\right)^2\left(\Delta - \delta\right)^4 + \left(Y_E - \Delta\right)^2 L^4} \right)\sin\beta\left(X_E, Y_E\right) + \frac{Y - Y_E}{\delta d}\cos\beta\left(X_E, Y_E\right) + \\ &+\left(1 - \frac{d}{\delta}\right)\left(\frac{\left(\Delta - \delta\right)^2\left(Y_E - \Delta\right)L^2}{\left(X_E - L\right)^2\left(\Delta - \delta\right)^4 + \left(Y_E - \Delta\right)^2 L^4} \right)\cos\beta\left(X_E, Y_E\right) - \frac{X - X_E}{\delta d}\sin\beta\left(X_E, Y_E\right) \end{aligned} \right\}_{D_4} \tag{7'}$$

4. Introduction of streamline function to verify the continuity equation

Because the mass conservation equation (4) is unstable in the iterative numerical calculus, we can it identically verify by introducing the streamline function $\Psi(X,Y)$ in the flow meridian plane, using the integrant factor $1/(R_o+X\sin\alpha +Y\cos\alpha)$ in the expressions of the both velocity components in the Cartesian trihedron:

$$U(X,Y)=\frac{1}{R_0+X\cos\alpha+Y\sin\alpha}\Psi'_Y \text{ and } V(X,Y)=\frac{-1}{R_0+X\cos\alpha+Y\sin\alpha}\Psi'_X . \quad (12)$$

By introducing these expressions and their partial differentials, we obtain the two equations with partial differentials to solve the proposed problem.

$$\frac{W'_X\Psi'_Y - W'_Y\Psi'_X}{R} + \frac{(\Psi'_Y\cos\alpha - \Psi'_X\sin\alpha)W}{R^2} = \nu\left[W''_{X^2} + W''_{Y^2} + \frac{W'_X\cos\alpha + W'_Y\sin\alpha}{R} - \frac{W}{R^2}\right] \quad (2')$$

5. Conclusions

Even if the solution will be obtained by numerical solving, the treated phenomenon is the only fluid flow case considering the molecular attraction forces of adhesion between the solid body and the liquid particles and cohesion between the liquid particles.

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$$
\begin{aligned}
&\frac{\Psi'_{\mathrm{Y}}}{R}\left(\begin{array}{l}\frac{1}{R}\Psi'''_{\mathrm{XY^2}}-\frac{2\cos\alpha}{R^2}\Psi''_{\mathrm{Y^2}}-\frac{\sin\alpha}{R^2}\Psi''_{\mathrm{XY}}+\frac{3\sin\alpha\cos\alpha}{R^3}\Psi'_{\mathrm{Y}}+\\ +\frac{1}{R}\Psi'''_{\mathrm{X^3}}-\frac{3\cos\alpha}{R^2}\Psi''_{\mathrm{X^2}}+\frac{3\cos^2\alpha}{R^3}\Psi'_{\mathrm{X}}\end{array}\right)+\\
&+\frac{\Psi'_{\mathrm{X}}}{R}\left(\begin{array}{l}\frac{3\sin\alpha}{R^2}\Psi''_{\mathrm{Y^2}}-\frac{3\sin^2\alpha}{R^3}\Psi'_{\mathrm{Y}}-\frac{3\sin\alpha\cos\alpha}{R^3}\Psi'_{\mathrm{X}}+\frac{2\sin\alpha}{R^2}\Psi''_{\mathrm{X^2}}-\\ -\frac{1}{R}\Psi'''_{\mathrm{Y^3}}-\frac{1}{R}\Psi'''_{\mathrm{X^2Y}}+\frac{\cos\alpha}{R^2}\Psi''_{\mathrm{XY}}\end{array}\right)+\\
&+\frac{2W}{R}\left(W'_{\mathrm{X}}\sin\alpha-W'_{\mathrm{Y}}\cos\alpha\right)= \qquad (7'')\\
&=\frac{\upsilon}{R}\left(\begin{array}{l}\Psi^{\mathrm{IV}}_{\mathrm{X^4}}+2\Psi^{\mathrm{IV}}_{\mathrm{X^2Y^2}}+\Psi^{\mathrm{IV}}_{\mathrm{Y^4}}-\frac{2\cos\alpha}{R}\Psi'''_{\mathrm{X^3}}-\frac{2\sin\alpha}{R}\Psi'''_{\mathrm{Y^3}}-\\ -\frac{2\sin\alpha}{R}\Psi'''_{\mathrm{X^2Y}}-\frac{2\cos\alpha}{R}\Psi'''_{\mathrm{XY^2}}+\frac{3\cos^2\alpha}{R^2}\Psi''_{\mathrm{X^2}}+\frac{3\sin^2\alpha}{R^2}\Psi''_{\mathrm{Y^2}}+\\ +\frac{6\sin\alpha\cos\alpha}{R^2}\Psi''_{\mathrm{XY}}-\frac{3\cos\alpha}{R^3}\Psi'_{\mathrm{X}}-\frac{3\sin\alpha}{R^3}\Psi'_{\mathrm{Y}}\end{array}\right)-\\
&-F_{\mathrm{c}}\left\{\begin{array}{l}\left(1-\frac{d}{\delta}\right)\left(\frac{-L^2\left(X_{\mathrm{E}}-L\right)\left(\Delta-\delta\right)^2}{\left(X_{\mathrm{E}}-L\right)^2\left(\Delta-\delta\right)^4+\left(Y_{\mathrm{E}}-\Delta\right)^2L^4}\right)\sin\beta\left(X_{\mathrm{E}},Y_{\mathrm{E}}\right)+\\ +\frac{Y-Y_{\mathrm{E}}}{\delta d}\cos\beta\left(X_{\mathrm{E}},Y_{\mathrm{E}}\right)-\frac{X-X_{\mathrm{E}}}{\delta d}\sin\beta\left(X_{\mathrm{E}},Y_{\mathrm{E}}\right)+\\ +\left(1-\frac{d}{\delta}\right)\left(\frac{\left(\Delta-\delta\right)^2\left(Y_{\mathrm{E}}-\Delta\right)L^2}{\left(X_{\mathrm{E}}-L\right)^2\left(\Delta-\delta\right)^4+\left(Y_{\mathrm{E}}-\Delta\right)^2L^4}\right)\cos\beta\left(X_{\mathrm{E}},Y_{\mathrm{E}}\right)\end{array}\right\}_{\mathrm{D_4}}
\end{aligned}
$$