# THE EFFICIENCY OF TWO-WINDING THREE-PHASE TRANSFORMERS IN HARMONIC AND UNBALANCED REGIME

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***Abstract**: This paper presents definitions for energy efficiency indicators of two-windings three-phase transformers, such as: active power efficiency* $\eta_P$, *active energy efficiency* $\eta_W$, *power losses* $\Delta P_{tr}$, *energy losses* $\Delta W_{tr}$, *active power own consume* $CPT_P$ *and active energy own consume* $CPT_W$ *for transformers from transforming stations (**ST**) and substations (**PT**).*

*A mathematical model is also presented, dedicated to the general case in which there is a transformer **ST/PT**, with presence of an unbalanced and non-symmetrical load.*

*Optimum electrical losses are highlited, due to the balanced and symmetrical load, as well as their variables, such as the variety of the loads fed from **ST/PT.***

**Keywords:** unbalance load, electrical efficiency, electrical power.

## 1. Introduction

The present socio-economic conditions allow the observation .of electrical losses types. Losses due to unbalance loads are between 8 and 15 % of overall consumed electrical energy. Under these circumstances, the determination of the specialists for creating an objective evaluation mechanism is highly motivated.

Energy losses in distribution networks can be split in two components: technical and commercial. Technical component has two sub-divisions (real and theoretical). Technical theoretical component can be minimized. It results that energy losses are an indicator which characterize the behavior of a specific electric network.

## 2. Informations needed for PT processing

For PT energy losses calculation, we need to know:

- the topology or electric scheme (fig 1);

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- specific characteristics of the transformers.

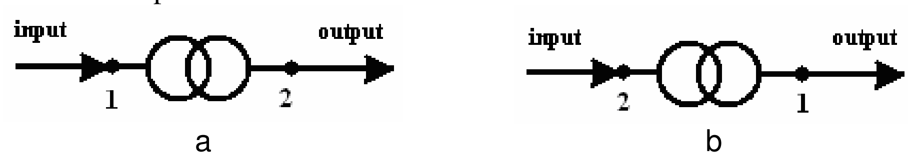



Fig. 1. PT electrical scheme

Rated parameters of PT transformers are: $S_n$ – rated power of T transformer; $\Delta P_o$ – empty regime losses of transformer; $\Delta P_{sc}$ – short-circuit losses of transformer; transforming ratio:

$$\frac{U_{n1}}{U_{n2}}\left(1 \pm p \cdot \frac{u_p}{100}\right) \quad (1)$$

for which: $U_{n1}$ is rated voltage for winding 1 of transformer T; $U_{n2}$ - rated voltage for winding 2 of transformer T; p – the active plot of the transformer T; $u_p$ – voltage variation on a transformer plot.

## 3. Consumer load curve

Under the hypothesis that data acquisition system has an interface which may calculate LV data: voltage U, current I, active power P or active energy $W_a$ for time intervals $\Delta t$, reactive energy Q or reactive energy $W_r$ for the same time intervals $\Delta t$, by using these data, load curves for each phase and for the whole system can be plotted.

Most used load curve indicators are:

- mean power $\mathbf{S_{med}}$:

$$S_{med} = \frac{\sqrt{W_a^2 + W_r^2}}{t_f} \quad (2)$$

- filling coefficient $\mathbf{k_u}$:

$$k_u = \frac{S_{med}}{S_M} \quad (3)$$

- factor form of load curve $\mathbf{k_f}$:

$$k_f = a + \frac{1-a}{k_u} \quad (4)$$

where: $\mathbf{S_M}$ is maximum power, **a** – regression coefficient between [0,15 ÷ 0,30], or $\mathbf{t_f}$ – operating time of consumer **C**.

## 4. Transformer power and energy losses

Starting from definitions of these associated values of single-phase transformers, for a symmetric three-phase transformer these become:

- Active power losses $\Delta$**P**

$$\Delta P = \Delta P_o + \Delta P_{sc} \cdot \frac{U_n^2}{S_n^2} \cdot (I_A^2 + I_B^2 + I_C^2) \quad (5)$$

- Active energy losses $\Delta$**W**$_a$

$$\Delta W_a = \left[ \Delta P_o \cdot t_f + \Delta P_{sc} \cdot \frac{U_n^2}{S_n^2} \cdot (I_A^2 \cdot k_A^2 + I_B^2 \cdot k_B^2 + I_C^2 \cdot k_C^2) \cdot t_f \right] \quad (6)$$

for which: $I_A$, $I_B$, $I_C$ are average operating currents of transformer; $k_A$ , $k_B$ , $k_C$ - form factors of load curves for each phase.

By considering feeding voltage to be symmetrical, and the currents being non-symmetrical and unbalanced, hypothesis which is valid for electrical distribution systems with/without ground wire, inverse non-symmetry coefficients can be defined, corresponding to the current $(k_I^o , k_I^-)$ as follows:

$$\underline{k}_I^- = \frac{\underline{I}^-}{\underline{I}^+} \text{ şi } \underline{k}_I^o = \frac{\underline{I}^o}{\underline{I}^+} \quad (7)$$

and sequence currents are

- **zero** (homopolar)

$$9 \cdot \left[I^o\right]^2 = \sum_{u \in F} I_m^2 + 2 \cdot \sum_{u,v \in F} I_u \cdot I_v \cdot \cos\varphi_{uv} \;\; ; u,v \in F \quad (8)$$

- **positive** (direct)

$$9 \cdot \left[I^+\right]^2 = \sum_{u \in F} I_m^2 + 2 \cdot \sum_{u,v \in F} I_u \cdot I_v \cdot \cos\left(\varphi_{uv} + \frac{4 \cdot \pi}{3}\right) \;\; ; u,v \in F \quad (9)$$

- **negative** (inverse)

$$9 \cdot \left[I^-\right]^2 = \sum_{u \in F} I_m^2 + 2 \cdot \sum_{u,v \in F} I_u \cdot I_v \cdot \cos\left(\varphi_{uv} + \frac{2 \cdot \pi}{3}\right) \;\; ; u,v \in F \quad (10)$$

$\varphi_{uv}$ angles belong to:

$$\varphi_{uv} \in \{\varphi_{AB} ; \varphi_{BC} ; \varphi_{CA}\}; u,v \in F \quad (11)$$

where **F** represents the transformer phases, and the angles $\varphi_{uv}$ are:

$$\varphi_{AB} = \varphi_A - \varphi_B ; \varphi_{BC} = \varphi_B - \varphi_C ; \; \varphi_{CA} = \varphi_C - \varphi_A \quad (12)$$

With upper notations, (5) becomes:

$$\Delta P = \Delta P_o + \Delta P_{sc} \cdot \left[1 + \left(k_I^-\right)^2 + \left(k_I^o\right)^2\right] \cdot \left(\frac{I^+ \cdot U_n}{S_n}\right)^2 \quad (13)$$

where $k_I^o$ and $k_I^-$ coefficients are defined by (7) taking into account (8÷10).

$\mathbf{U^+ = U_n}$ and $\mathbf{I^+}$ are positive sequence components of voltage and current respectively, the latter one being defined by (9).

If form factor $k_f^2$ is defined as:

$$k_f^2 = \frac{I_A^2 k_A^2 + I_B^2 k_B^2 + I_C^2 k_C^2}{I_A^2 + I_B^2 + I_C^2} \tag{14}$$

then (6) becomes:

$$\Delta W_a = \left[ \Delta P_o + \Delta P_{sc} \cdot \frac{U_n^2}{S_n^2} \cdot k_f^2 \cdot (I_A^2 + I_B^2 + I_C^2) \right] \cdot t_f \tag{15}$$

which, by taking into account non-symmetry coefficients ($k_I^o$, $k_I^-$) and positive sequence components expressions from (12) leads to:

$$\Delta W_a = \left\{ \Delta P_o + \Delta P_{sc} \cdot \left( \frac{U^-}{S_n} \right)^2 \cdot \left[ 1 + \left( k_I^- \right)^2 + \left( k_I^o \right)^2 \right] \cdot k_f^2 \cdot \left( I^+ \right)^2 \right\} \cdot t_f \tag{16}$$

If we note with $\mathbf{S^+}$ as follows

$$S^+ = 3 \cdot U^+ \cdot I^+ \tag{17}$$

which is defined as positive sequence apparent power and we introduce load coefficient in apparent power $\alpha_S$ defined as:

$$\alpha_S = \frac{S^+}{S_n} \tag{18}$$

then the relations (13) and (15) become:

$$\begin{aligned} \Delta P &= \Delta P_o + \Delta P_{sc} \cdot \left[ 1 + \left( k_I^- \right)^2 + \left( k_I^o \right)^2 \right] \cdot \alpha_S^2 \\ \Delta W_a &= \left\{ \Delta P_o + \Delta P_{sc} \cdot \left[ 1 + \left( k_I^- \right)^2 + \left( k_I^o \right)^2 \right] \cdot k_f^2 \cdot \alpha_S^2 \right\} \cdot t_f \end{aligned} \tag{19}$$

Under the hypothesis of a total compensation of power factor, $\lambda = 1$ and if we define

- power efficiency $\eta_P$

$$\eta_P = \frac{P_2}{P_1} \tag{20}$$

- energy efficiency $\eta_W$

$$\eta_W = \frac{W_2}{W_1} \tag{21}$$

By taking into account the significance of power from figure 1, by replacing active power and active energy losses from (19), efficiency expressions become:

- power efficiency $\eta_P$

$$\eta_P = \frac{P_2}{P_2 + \Delta P} = \frac{P_2}{P_2 + \Delta P_o + \Delta P_{sc} \cdot \left[1 + \left(k_I^-\right)^2 + \left(k_I^o\right)^2\right] \cdot \left(\frac{P_2}{S_n}\right)^2} \tag{22}$$

- energy efficiency $\eta_W$

$$\eta_W = \frac{W_2}{W_2 + \Delta W} = \frac{W_2}{W_2 + \Delta P_o \cdot t_f + \Delta P_{sc} \cdot \left[1 + \left(k_I^-\right)^2 + \left(k_I^o\right)^2\right] \cdot \left(\frac{W_2}{S_n \cdot t_f}\right)^2 \cdot k_f^2 \cdot t_f} \tag{23}$$

If we choose as variable

$$\alpha_P = \frac{P_2}{S_n} \text{ and } \alpha_W = \frac{W_2}{S_n \cdot t_f} \tag{24}$$

then the efficiencies (22) and (23), respectively, become:

- power efficiency $\eta_P$

$$\eta_P = \frac{\alpha_P}{\alpha_P + \frac{\Delta P_o}{S_n} + \frac{\Delta P_{sc}}{S_n} \cdot \left[1 + \left(k_I^-\right)^2 + \left(k_I^o\right)^2\right] \cdot \alpha_P^2} \tag{25}$$

- energy efficiency $\eta_W$

$$\eta_W = \frac{\alpha_W}{\alpha_W + \frac{\Delta P_o}{S_n} + \frac{\Delta P_{sc}}{S_n} \cdot \left[1 + \left(k_I^-\right)^2 + \left(k_I^o\right)^2\right] \cdot k_f^2 \cdot \alpha_W^2} \tag{26}$$

$\alpha_P$ and $\alpha_W$ values for which (25) are (26) maximum, are:

- for $\alpha_P$

$$\alpha_P = \sqrt{\frac{\Delta P_o}{\Delta P_{sc}}} \cdot \frac{1}{\sqrt{1 + \left(k_I^o\right)^2 + \left(k_I^-\right)^2}} \tag{27}$$

- for $\alpha_W$

$$\alpha_W = \sqrt{\frac{\Delta P_o}{\Delta P_{sc}}} \cdot \frac{1}{k_f \cdot \sqrt{1 + \left(k_I^o\right)^2 + \left(k_I^-\right)^2}} \tag{28}$$

and the maximum of each function from (25) and (26) is:

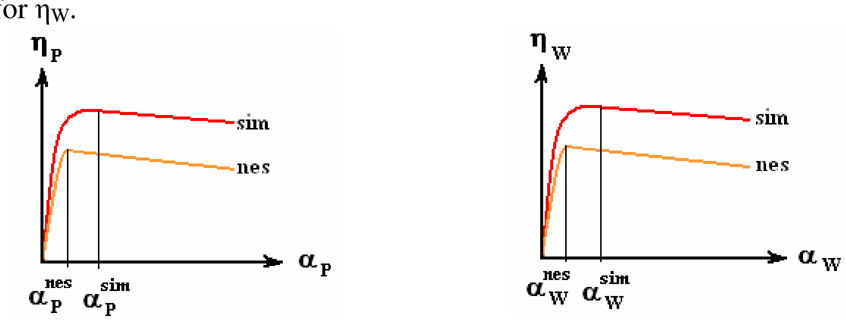
- efficiency $\eta_P$

$$\eta_P = \frac{S_n}{S_n + 2 \cdot \sqrt{\Delta P_o \cdot \Delta P_{sc}} \cdot \sqrt{1 + \left(k_I^o\right)^2 + \left(k_I^-\right)^2}} \tag{29}$$

- efficiency $\eta_W$

$$\eta_W = \frac{S_n}{S_n + 2 \cdot k_f \cdot \sqrt{\Delta P_o \cdot \Delta P_{sc}} \cdot \sqrt{1 + \left(k_I^o\right)^2 + \left(k_I^-\right)^2}} \tag{30}$$

The dependencies $\eta_P = f(\alpha_P)$ and $\eta_W = f(\alpha_W)$ are presented in figure 2:
a) for $\eta_P$;
b) for $\eta_W$.




a. efficiency $\eta_P$ b. efficiency $\eta_W$

Figure 2. Efficiencies $\eta_P$ (a) and $\eta_W$ (b) in symmetric and non-symmetric regime

## 5. Conclusions

By analyzing the curves from figure 2 results some observations which allows identifying some measures for PT use efficiency upgrading. Some of these mesures are relevant:

- correct choose of transformer parameters ($\Delta P_o$ şi $\Delta P_{sc}$) in order to be ensured the need of the consumers with minim losses (power and energy);
- ensuring of the symmetric load regime $k_I^o$ and equilibrated $k_I^-$ which allows, for the same transit of electric energy, high efficiency and therefore minimum power and energy losses;
- adequate tarifary measures for a better aplatization of load curves which, in symmetric (and non-symmetric) regimes, leads to smaller power and energy losses.

These elements are known and presented in all current standards which must be filled with studies regarding load curves which will allow the creation of new reglementations in electricity tariffs domain.

## 6. Case study

By using the upper presented model, a numerical application has been created for determining power and energy losses for a PT of which characteristics are presented in table 1.

Table 1. Equipment and load characteristics for 16kVA transformer

| | | | |
|---|---|---|---|
| $U_{np}$ / $U_{ns}$ [kV]/ [kV] | 20 / 0,4 | $I_n$ [A] | 40,0 |
| $\Delta P_o$ [kW] | 0,157 | $\Delta P_{sc}$ [kW] | 0,856 |

Using the relations upper mentioned relations, the values from table 2 are obtained, which highlight the unbalanced load of transformer for which this analysis has been performed.

Table 2. Non-symmetric loads of 16kVA transformer

| $I_A$ | $I_B$ | $I_C$ | $\varphi_A$ | $\varphi_B$ | $\varphi_C$ | $I^o$ | $I^-$ | $I^+$ | $k_I^o$ | $k_I^-$ |
|---|---|---|---|---|---|---|---|---|---|---|
| A | A | A | o | o | o | A | A | A | | |
| 2,0 | 2,0 | 2,0 | 0 | 240 | 120 | 0,0 | 0,0 | 2,0 | 0,000 | 0,002 |
| 8,0 | 6,5 | 5,0 | 3 | 246 | 129 | 0,9 | 0,9 | 6,5 | 0,141 | 0,133 |
| 14,0 | 11,0 | 8,0 | 6 | 252 | 138 | 2,0 | 1,7 | 11,0 | 0,178 | 0,159 |
| 20,0 | 15,5 | 11,0 | 9 | 258 | 147 | 3,2 | 2,7 | 15,4 | 0,205 | 0,175 |
| 26,0 | 20,0 | 14,0 | 12 | 264 | 156 | 4,6 | 3,7 | 19,7 | 0,232 | 0,189 |
| 32,0 | 24,5 | 17,0 | 15 | 270 | 165 | 6,2 | 4,9 | 24,0 | 0,260 | 0,204 |
| 36,0 | 27,5 | 19,0 | 17 | 274 | 171 | 7,5 | 5,7 | 26,8 | 0,280 | 0,215 |

Power and energy losses, with respect to the non-symmetry coefficients, for 16kVA PT , is presented in fig.3 and fig. 4.

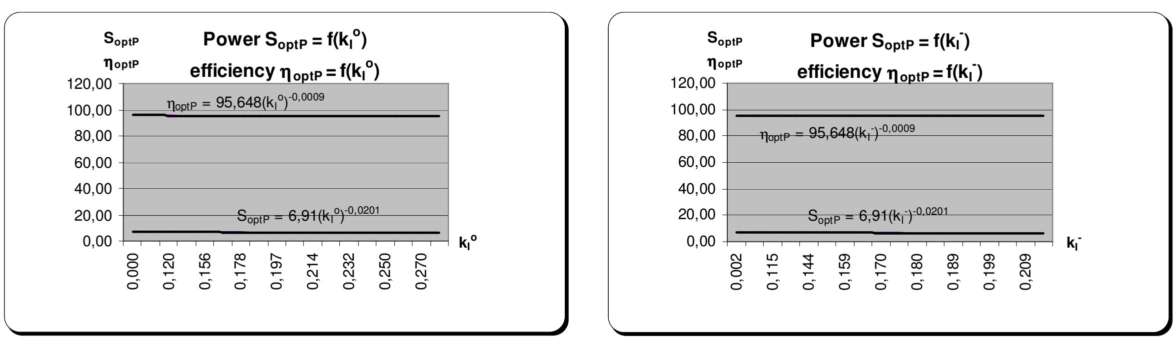



Fig. 3. Apparent power $\mathbf{S_{optP}}$ and efficiency $\boldsymbol{\eta_{optP}}$ variation with respect to non-symmetry coefficients

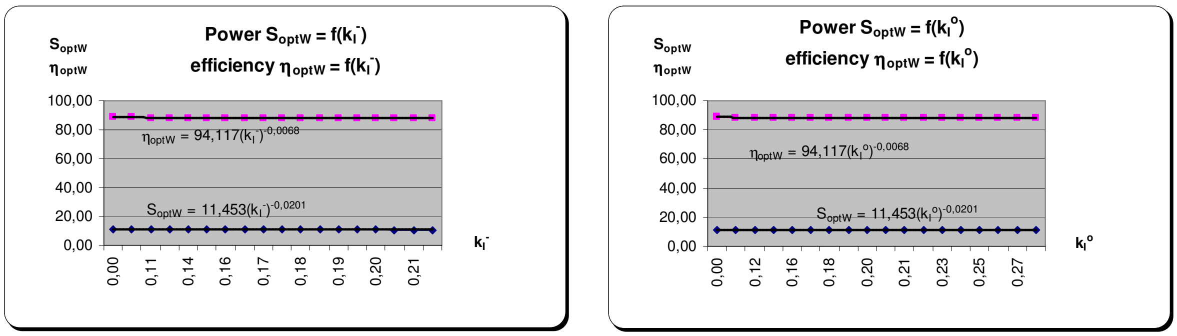



Fig. 4. Apparent power $\mathbf{S_{optW}}$ and efficiency $\boldsymbol{\eta_{optW}}$ variation with respect to non-symmetry coefficients

Power and energy losses variation, with respect to the load factor is no longer graphically presented, due to the fact that being a very reduced non-symmetric regime, the characteristics of the losses from symmetric regime will overlap over non-symmetric regime one.

For this case analysis, the dependencies of energy losses for various $k_f^2$ form factors have been achieved, for various values of power factor.

## 7. Results interpretation

The results from the latter paragraph have been obtained by a specialized calculation program, created by authors.

Such a technical and economic model is useful for creation of technical and managerial measures, financial strongholds which may validate and justify the strategy of making electrical energy distribution economically profitable.

Thus, for increasing economic efficiency of electrical energy distribution, a few technical measures are recommended in current literature, such as:

- correct placement of harmonic filters – technical measures for damping the disturbed regime;
- technical and economic sizing of harmonic filters.

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