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MULTI-CRITERIA ANALYSIS FOR OPTIMAL PLACEMENT OF DISPERSED GENERATORS IN DISTRIBUTION ELECTRIC NETWORKS

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The paper presents an analysis on the optimal placement of dispersed generators within the distribution networks. The objective function is multi-criteria and consists of the cost of power and energy losses in a distribution network as well as the cost of customer interruptions. By minimization of this function, an optimal number of dispersed generators as well as their rated powers are achieved. The constraints taken into account in this optimisation problem is related to the nodal voltage level and the thermal limit of the network branches. A medium voltage distribution network is used to test the proposed optimisation model.

Keywords: distribution networks, dispersed generation, power losses, energy losses, reliability.

1. Introduction

Distributed generation refers to local generation within consumption areas of relatively small amounts of power with respect to the classical power plants (thermal, nuclear, hydro, etc.). The differences of the distributed generation sources with respect to the classical ones are the location and size. The small rated *DG* sources are, in general, connected to the distribution networks. In some cases they represent not only a supply power reserve for the large transmission networks but also a back-up of the classical power plants.

Dispersed Generators (*DGs*) can lead to various changes in the distribution networks operation to which they are connected. Among these it should be mentioned: currents/powers flows in the network branches, leading to changes in the power and energy losses as well as in the voltage drops and nodal voltages; reliability indices concerning the security of the supply service; voltage and current waveform.

Depending on the amount of the size, location and technology used the *DGs* can affect favourable or unfavourable the network operation and therefore can influence the load supply service.

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The interest of engineers for distributed generation regards various issues related to the evaluation of their impact and the improvement of electrical networks operation and therefore the quality of supply service. Therefore, in paper [1] the effect of voltage control on the losses in the network with distributed generation is analyzed. Paper [2] makes and evaluation of the *DG* impact on the reliability of consumers supplying paths, while in paper [3] the losses reduction in the distribution electrical networks by reconfiguration in the presence of *DG* sources is analyzed. Other issue related to the penetration of distribution generation in the distribution networks is the losses allocation, debated in [4] and [5]. In paper [6] the optimal placement of distributed generators using the fuzzy logic and genetic algorithms for power losses costs reduction is also analyzed. The optimal placement problem of the *DG* sources, under various conditions, is also studied in [7] and [8].

Structural and operational transformations of the actual power systems have created a competitive framework where the economic aspects are of increased importance. In this context, the economical and reliable operation of power systems becomes primordial.

Our paper deals with optimal placement of *DG* for operational costs reduction, represented by the cost of power and energy losses and the cost of customer interruptions.

2. Mathematical model

The mathematical model of the *DGs* placement problem consists of the objective function and constraints.

A. Objective Function

The mathematical expression of the objective function can be written under the form:

$$\text{MIN } \left[LC + CIC\right] \tag{1}$$

where *LC* is the losses cost for network branches and *CIC* represents the customer interruption cost for all consumption nodes.

B. Constraints

For this optimization problem the following constraints are considered:

1) *Power Flow Equations* [6]:

$$F_i\left(\mathbf{x}, P_{G,k}\right) \quad i = 1,\ 2, \ldots, n \tag{2}$$

where F_i is the active and reactive power balance equations for the consumption node *i*, $\mathbf{x}$ is the vector of state variables for all nodes of the electrical network and $P_{G,k}$ is the dispersed generators control variable (active power) at the node *k*.

2) *Operational constraints*:

$$\begin{aligned} & V_i^{\min} \le V_i \le V_i^{\max} \\ & P_l \le P_l^{\max} \end{aligned} \tag{3}$$

where $V_i^{\min}$ and $V_i^{\max}$ are the admissible limits for the voltage level V_i in all nodes *i*, $P_l^{\max}$ is the load-transfer capability of all branches *l* of the network.

3) *Constraints on the size and number of DGs*:

$$\begin{aligned} & P_{G,k}^{\min} \le P_{G,k} \le P_{G,k}^{\max} \\ & n_{DG} \le n_{DG}^{\max} \end{aligned} \tag{4}$$

where $P_{G,k}$ is available active rated power chosen between certain limits $P_{G,k}^{\min}$ and $P_{G,k}^{\max}$, k denotes the node's number and $n_{DG}^{\max}$ is the maximum number of the *DGs* from the network.

3. Evaluation of objective function

The objective function is defined by combining two criteria: losses cost *LC* and customer interruption cost *CIC*. The quantitative and qualitative properties of the two criteria are presented in the following.

A. Evaluation of losses cost

The power losses can be identified by several components, of which the most important is given by the technical losses. Two components can be identified in this category: power losses and energy losses.

The active power losses for a three-phase electrical line *l*, of resistance R_l, are given by:

$$\Delta P_l = 3 R_l I_l^2 \tag{5}$$

where I_l is the amplitude of the current flowing on the line.

The energy losses on the electrical line *l* are obtained by integrating in time the power losses:

$$\Delta W_l = 3 R_l \int_0^T i^2(t)\, dt \tag{6}$$

where $i(t)$ indicates the variation curve of the line current, and *T* is the time duration for which the energy losses are calculated.

Usually, variation curve of the line current in time is difficult to find, so that in order to evaluate the power losses various methods are used. The most used methods are based on the losses time, case in which the calculation time is large. For this reason, in our work we have applied the energy summation method [9]. A process identical to the backward sweep obtains energy losses as in the case of the backward-forward method applied for power flow calculation. The difference between the two methods is that the first one uses average complex powers of the loads:

$$\underline{S}_m = P_m + jQ_m = K_{P\max}\left(P_{\max} + jQ_{\max}\right) \tag{7}$$

where $P_{\max}$ and $Q_{\max}$ are the components of the maximum power, and $K_{P\max}$ is a coefficient of maximum power use.

By applying the backward sweep, considering a *m* average power $\underline{S}_m$ for every load, we obtain the power flow and the power losses on the network lines. Denoting by $\Delta P_{m,l}$ the active power losses through the electrical line *l*, produced in the time interval *T*, the energy losses are given by:

$$\Delta W_l = \Delta P_{m,l} \cdot T \tag{8}$$

For the evaluation of the losses cost, the per kilowatt cost of power losses (c_{PL}) and the per kilowatt-hour cost of the energy losses (c_{WL}) are taken into account, so that [10]:

$$LC_l = c_{PL}\Delta P_l + c_{WL}\Delta W_l \tag{9}$$

The losses cost *LC* for all network branches is:

$$LC = \sum_l LC_l \tag{10}$$

B. Evaluation of customer interruption cost

The interruption cost for a consumer *i* is evaluated by considering the per kilowatt cost of the interrupted power (c_P) and the per kilowatt-hour cost of the energy not supplied (c_W):

$$IC_i = \lambda_{ei}\left[c_P\left(r_{ei}\right) + c_W\left(r_{ei}\right) r_{ei}\right]\Delta L_i \tag{11}$$

where λ_{ei} is the equivalent failure rate, r_{ei} is the equivalent interruption duration of supply and ΔL_i is the interrupted power at the consumer *i*.

The quantities c_P and c_W depend on the equivalent interruption duration r_{ei} and type of the consumption node *i* (residential, commercial or industrial). The equivalent reliability indices λ_{ei} and r_{ei} of the consumption node *i* are calculated according to [7].

The customer interruption cost *CIC* for all *n* consumption nodes is obtained by:

$$CIC = \sum_{i=1}^{n} IC_i \quad (12)$$

The introduction of dispersed generation in electrical distribution networks can have a favourable influence on the safe of consumers' supply, if some conditions are met:

– the operation of the dispersed sources should be reliable and should not depend on the environment conditions (presence of wind, of solar radiations, etc.);
– the existence into the electrical network of some switching equipments (circuit breakers) capable to automatic isolate the supplied area for a fault in any point from the network;
– the possibility of islanded operation of the dispersed sources (the existence of voltage and frequency regulators).

For the shake of simplicity, it was assumed that each dispersed source could supply only the consumer from the node to which it is connected. The radial network from figure 1 was considered, where a dispersed source is connected to the node *k*, which cover totally or partially the consumption of the node. The separation of the area supplied by a dispersed source is performed by means of circuit breakers B_2 and B_3. The presence of the dispersed source and the afferent circuit breakers modifies the reliability indices of the *k* node, as well as of others nodes from the area. The analysis of the failure type of the network with respect to the node *k* is presented in [7].

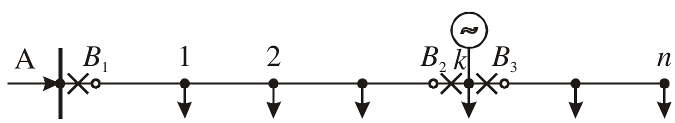

Fig. 1. Simple radial electrical network with one *DG*.

Considering that the generator connected at the node *k* has the rated power $P_{G,k}$ and the active power consumed at the node *k* is P_k, the interruption cost of this node becomes:

$$\begin{aligned} IC_k = \; & \lambda_{ek}\left[c_P(r_{ek}) + c_W(r_{ek})\,r_{ek}\right]\left(P_k - P_{G,k}\right) + \\ & + \lambda_{ek}\left[c_P(r'_{ek}) + c_W(r'_{ek})\,r'_{ek}\right]P_{G,k} \end{aligned} \quad (13)$$

where r'_{ek} is the interruption of the DG_k operations duration during the interruption duration r_{ek} of the supply from the system, taking into account the availability $p_{DG,k}$ of this *DG*:

$$r'_{ek} = \left(1 - p_{DG,k}\right) r_{ek} \quad (14)$$

4. Solving of mathematical model

The mathematical model described by the relationships (1)÷(4) represents the formulation of the mathematical programming problem. The unknowns of the problem are: the number of the dispersed sources from the network (n_{DG}) and the power generated by each source ($P_{G,k}$). For a given configuration of the electrical network, the equivalent reliability indices of a consumption node are independent of the indices of other nodes. Under these conditions, based on relation (13), it results that the optimal active power $P_{G,k}^{\text{opt}}$ that should be generated at the node k must be equal to consumed active power P_k at this node. The *DGs* number n_{DG} and optimal placement are to be further determined. The solution manner is based on searching into the solutions space. The use of searching heuristic methods is intricate and therefore an exhaustive searching is used. Thus, consider only one DG_k that is placed by turn in every consumption node ($k = 1, 2, \ldots, n$), using the optimal value of the rated active power $P_{G,k}^{opt}$ for each generator. The optimal placement is retained, for which *CIC* has the lowest value.

Next, consider two sources DG_j and DG_k ($j, k = 1, 2, \ldots, n;\ j \neq k$), and the previous algorithm is repeated, retaining the optimal solution. This proceeding goes on until all $n_{DG}^{\max}$ are verified.

5. Case study

The testing of multi-criteria analysis proposed in this paper has been made on the distribution network with 33 nodes and 32 branches from [9]. Figure 2 presents the radial configuration was for the study case. The resistance and the reactance of each branch as well as the active and reactive powers in the load busses are the same with the ones presented in [9]. The lengths of the line sections and the $K_{P\max}$ coefficient for the consumption points are presented in Table I.

We consider the existence of one circuit breaker on the feeder out from the supply node 0 (on the branch 0-1 at the node 0), and at the others nodes there exists sectionalisers. The reliability parameters used in calculations are the ones given in Table II.

The values of interruption costs used in calculations are: $c_P = 5$ €/kW and $c_W = 1$ €/kWh. For the evaluation of the losses cost, the values are: $c_{PL} = 10$ €/kW and $c_{WL} = 0.2$ €/kWh. The study period T is taken on a year basis

($T = 8760$ hours).The availability of each DG is considered $p_{DG} = 0.6$. The maximum number of DGs has been considered $n_{DG}^{max} = 2$.

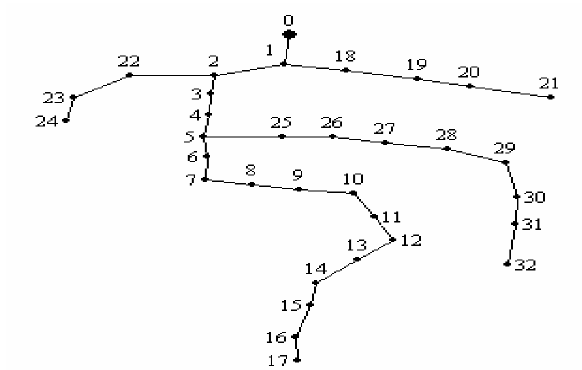

Fig. 2. Electrical network with 33 buses and 32 branches.

Table 1

Data for the test electrical network

Element		Length	K_{Pmax} at receiving node	Element		Length	K_{Pmax} at receiving node
Sending end	Receiving end	m		Sending end	Receiving end	m	
0	1	220	0,4	16	17	1270	0,3
1	2	1180	0,3	1	18	380	0,3
2	3	440	0,4	18	19	3440	0,3
3	4	460	0,3	19	20	1330	0,3
4	5	1870	0,3	20	21	2300	0,3
5	6	960	0,5	2	22	780	0,3
6	7	600	0,5	22	23	1550	0,5
7	8	1780	0,3	23	24	1550	0,5
8	9	1810	0,3	5	25	240	0,3
9	10	170	0,3	25	26	340	0,3
10	11	320	0,3	26	27	2420	0,3
11	12	2540	0,3	27	28	1840	0,4
12	13	1760	0,4	28	29	610	0,5
13	14	1350	0,3	29	30	2230	0,4
14	15	1290	0,3	30	31	1010	0,5
15	16	4190	0,3	31	32	1390	0,3

Table 2

Component reliability data

Index / Component	Failure rate $\lambda[\text{f/yr}]$	Repair time $r_{rep}[\text{h}]$	Isolation time $r_{isol}[\text{h}]$
Line [1 km]	0.045	8	2
Circuit breaker	0.036	16	2
Switch	0.003	17	2

The cost values for the initial case (without *DGs*) are *LC* = 64144 €/kW and CIC = 54271 €/kW. For the shake of simplicity, the costs associated to the configurations where *DG* sources were introduced are referred to the values obtained in the initial case. Therefore, the variation of CIC with the introduction of one *DG* in each load node is shown in figure 3. From this figure it can be seen that the bus 5 with the smallest value of the total cost.

The costs variation to simultaneous introduction of two *DGs* into the electrical network is shown in figure 4, in increasing order for the first 10 combinations. The optimal placement is obtained for nodes 5 and 18. 496 configurations were analyzed.

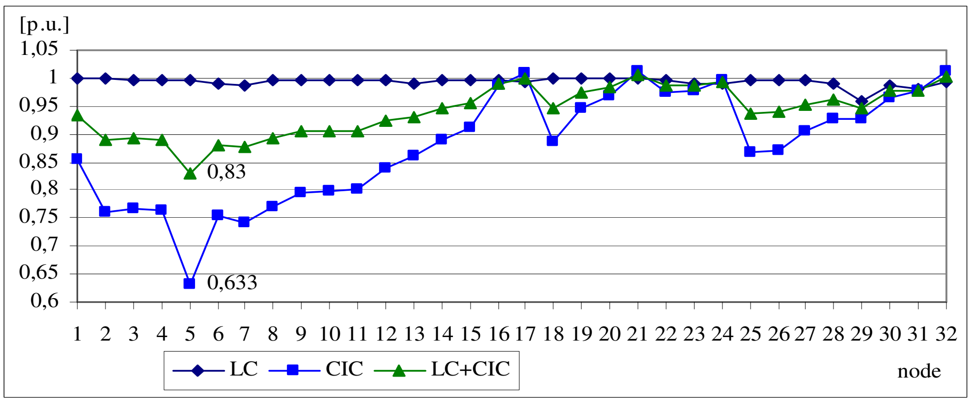

Fig. 3. Costs variation (in p.u.) for one *DG*.

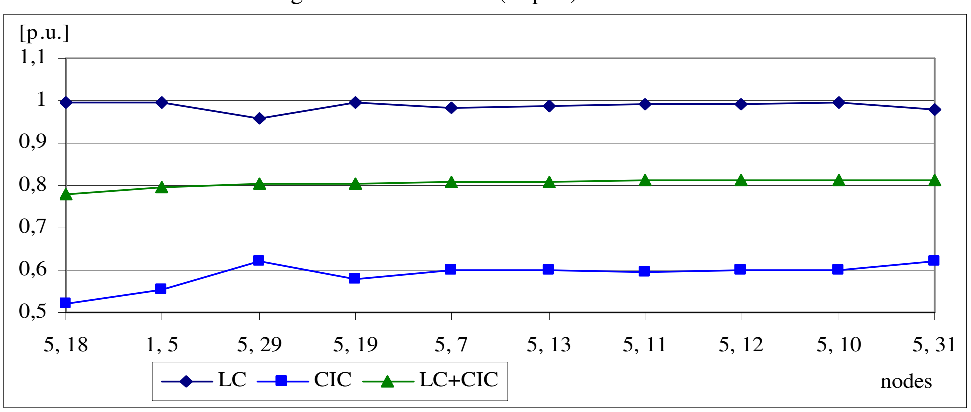

Fig. 4. Costs variation (in p.u.) for two *DGs*.

In the previous analysis it was assumed that the optimal generated power for each bus is equal to the load power. Under these conditions it is difficult to establish the optimal bus because the nodal generated powers are different. In order to establish which of the nodes gives the maximum gain, the case with uniform load for all consumers, that is $\underline{S} = (116.1 + j71.9)\,\text{kVA}$, was considered.

The average value 0.40 was considered for $K_{P\max}$ coefficient. The costs values for the initial case (without *DGs*), in the case of uniform load are *LC* = 70298 €/kW and CIC = 54633 €/kW. Figures 5 and 6 shows the results obtained for one *DG* and two *DGs*, respectively.

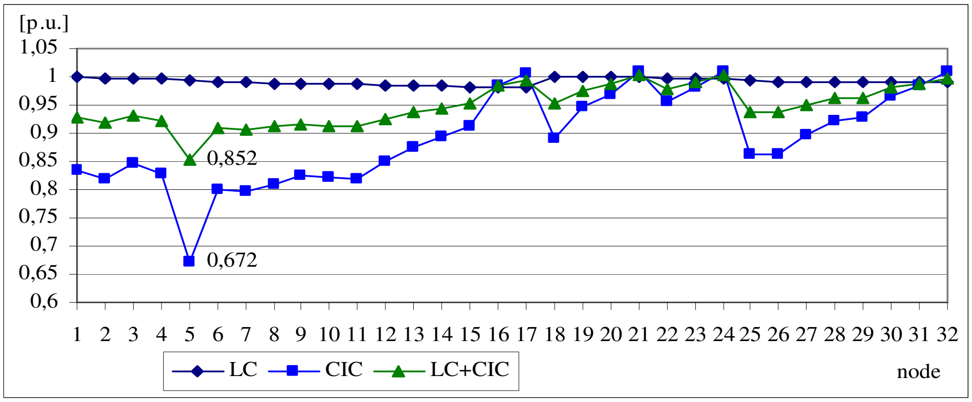

Fig. 5. Costs variation (in p.u.) for one *DG* considering uniform consumption at nodes.

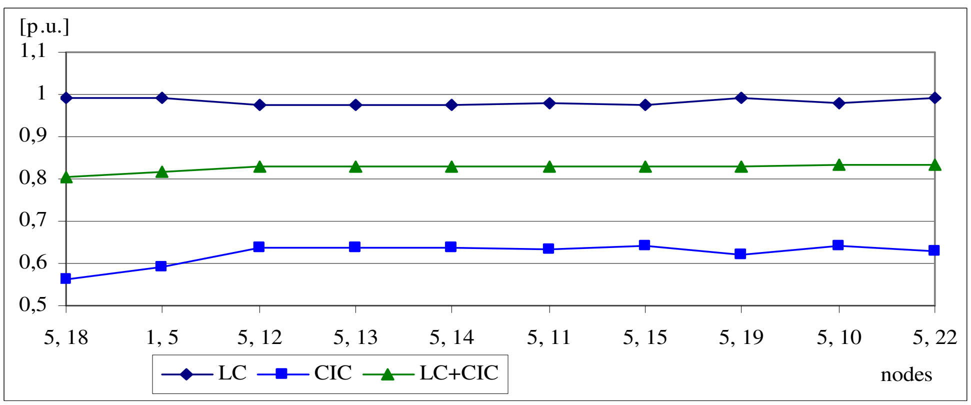

Fig. 6. Costs variation (in p.u.) for two *DGs* considering uniform consumption at nodes.

6. Conclusions

The optimal placement of distributed generators in the distribution networks, by simultaneous considering the costs of power and energy losses and the costs of load interruption, was studied. The characteristics of the two issues are quite different, in the sense that obtaining an optimum for one issue does not mean that it represent the optimum for the other issue.

The losses costs reduction is more important when the connection point of the *DG* is electrically far from the radial network sources (supplying bus). When considering the interruption cost criterion, the optimal placement location is situated in the middle of the radial network.

The interruption cost reduction is due to the introduction of the *DGs* which ensure the continuity in the load supply when a fault occurs in the network but also due to presence of the circuit breakers on both sides bus where the distributed generator is connected, of which purpose is to automatically insulate the part of the network supplied by the *DG*, when a fault occurs in the network.

The present work considered the back-up possibility, in the case of a fault in the distribution network, only of the load supplied from the bus where the *DG* is connected. A more complex study could focus on establishing the optimal supply area in the case if a fault.

A radial configuration of a meshed network was considered in our study case. For such networks, the benefits gained by introducing *DGs* could be combined with the advantages offered by reconfiguration.

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